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Hidden Color and the b_1 structure function of the deuteron.

Received: date / Accepted: date

Abstract The b_1 structure function is an observable feature of a spin-1 system sensitive to non-nucleonic components of the target nuclear wave function. A simple model for hidden-color, six-quark configurations is proposed and found to give substantial contributions for values of $x > 0.2$. Good agreement with Hermes data is obtained. Predictions are made for an upcoming JLab experiment.

Keywords Deep inelastic scattering · tensor polarized target

1 Introduction

This document is based on the paper [17]. That paper considered several different effects that contribute to the b_1 structure function of the deuteron. Here only the effects of hidden-color components of the deuteron are discussed.

Deep inelastic scattering from a spin-one target has features, residing in the leading-twist b_1 structure function, that are not present for a spin-1/2 target [13; 8]. In the Quark-Parton model

$$b_1 = \sum_q e_q^2 \left[q_{\uparrow}^0 - \frac{1}{2}(q_{\uparrow}^1 + q_{\uparrow}^{-1}) \right] \equiv \sum_i e_q^2 \delta q_i \quad (1)$$

where $q_{\uparrow}^n(q_{\downarrow}^m)$ is the number density of quarks with spin up(down) along the z axis in a target hadron with helicity m . The function b_1 is called the tensor structure function of the deuteron because it has been observed by the Hermes collaboration using a tensor polarized deuteron target [2] for values of Bjorken $0.01 < x < 0.45$. The function b_1 takes on its largest value of about 10^{-2} at the lowest measured value of x (0.012), decreases with increasing x through zero and takes on a minimum value of roughly -4×10^{-3} .

The function b_1 nearly vanishes if the spin-one target is made of constituents in a relative s -state, and is very small for a target of spin 1/2 particles moving non-relativistically in higher angular momentum states [13; 14; 21]. Thus one expects [13] that a nuclear b_1 may be dominated by non-nucleonic components of the target nuclear wave function. Consequently, a Jefferson Laboratory experiment [19] is planned to measure b_1 for values of x in the range $0.16 < x < 0.49$ and $1 < Q^2 < 5 \text{ GeV}^2$ with the aim of reducing the error bars. Our focus here is on the kinematic region of the largest higher values of x that are available to the JLab experiment. The Hermes experimental result [2] presents an interesting puzzle because it observed a significant negative value of b_1 for $x = 0.45$. At such a value of x , any sea quark effect such as arising from double-scattering or virtual pions is completely negligible. Furthermore, the nucleonic contributions are computed to be very small [14; 21], so one must consider other possibilities. We therefore take up the possibility that the deuteron has a six-quark component

that is orthogonal to two nucleons. Such configurations are known to be dominated by the effects of so-called hidden-color states in which two color-octet baryons combine to form a color singlet [11]. Such configurations can be generated, for example, if two nucleons exchange a single gluon leading to a quantum fluctuation involving an color octet and color anti-octet baryon.

In particular, a component of the deuteron in which all 6 quarks are in the same spatial wave function ($|6q\rangle$) can be expressed in terms on nucleon-nucleon NN , Delta-Delta $\Delta\Delta$ and hidden color components CC as [11]:

$$|6q\rangle = \sqrt{1/9}|N^2\rangle + \sqrt{4/45}|\Delta^2\rangle + \sqrt{4/5}|CC\rangle. \quad (2)$$

This particular state has an 80% probability of hidden color and only an 11% probability to be a nucleon-nucleon configuration. The 80 % cited here is a purely algebraic number that applies only for completely overlapping nucleons. The real question is the probability that the deuteron consists of 6 quarks are in the same spatial wave function, which is denoted here as P_{6q} . A recent review of hidden color phenomena is presented in [5]. In the following, the term $|6q\rangle$ is referred to interchangeably as either a six-quark or hidden color state.

The discovery of the EMC effect [4] caused researchers to consider the effects of such six-quark states [7] in a variety of nuclear phenomena [16; 15; 18]. Furthermore, the possible discovery of such a state as a di-baryon resonance has drawn recent interest [6]. Therefore we propose a model of a hidden-color six-quark components of the s and d -states of the deuteron. We also note that including a six-quark hidden color component of the deuteron does not lead to a conflict with the measured asymptotic d to s ratio of the deuteron [10]. The EMC effect remains the only nuclear effect that has not been explained using conventional (non-quark) dynamics [3; 9; 12].

2 Hidden-color six-quark states

We investigate the possible relevance of hidden-color six-quark states. There have been many models and attempts to provide definitive evidence for the existence of such states. However, no model has been unambiguously and uniquely verified by experiment. This is because conventional nuclear theory makes no reference to such states and yet successfully reproduces all known nuclear phenomena except for the EMC effect. In this paper, we take the view that the HERMES observation at $x = 0.452$ (their largest value of x) may provide another example requiring unconventional nuclear wave functions, and therefore may offer an opportunity to finally learn something definite. The key point is that nucleonic (and mesonic) effects, as presently computed, offer much smaller (in magnitude) values of b_1 than found by HERMES. A value of $x = 0.452$ involves valence quarks. Since nucleons do not provide a mechanism, one is naturally encouraged to look at six quark hidden color components, which should have support at such values of x . Since no definitive model exists, it is sufficient to use the simplest of many possible models for the present exploratory calculation.

We proceed by assuming the existence of a deuteron component consisting of six non-relativistic quarks in an S -state. As stated in the Introduction, such a state has only a probability of 1/9 to be a nucleon-nucleon component, and is to a reasonable approximation a hidden color state, so we use the terminology six-quark, hidden color state. Then we obtain the corresponding d state component by promoting any one of the quarks to a $d_{3/2}$ -state. We define these states by combining 5 s -state quarks into a spin 1/2 component, which couples with either the $s_{1/2}$ or $d_{3/2}$ single-quark state to make a total angular momentum of 1. We therefore write the wave functions of these states for a deuteron of $J_z = H$ as

$$\psi_{j,l,H}(\mathbf{p}) = \sqrt{N_l} f_l(p) \sum_{m_s, m_j} \mathcal{Y}_{jlm_j} \langle jm_j, \frac{1}{2}m_s | 1H \rangle, \quad (3)$$

where $l, j = s_{1/2}$ or $d_{3/2}$, N_l is a normalization constant chosen so that $\int d^3p \bar{\psi}_{j,l,H}(\mathbf{p}) \gamma^+ \psi_{j,l,H}(\mathbf{p}) = 1$ and \mathcal{Y}_{jlm_j} is a spinor spherical harmonic. The matrix element for transition between the $l = 0$ and $l = 2$ states is given by the light-cone distribution:

$$F_H(x_{6q}) = \frac{1}{2} \int d^3p \bar{\psi}_{1/2,0,H}(\mathbf{p}) \gamma^+ \psi_{3/2,2,H}(\mathbf{p}) \delta\left(\frac{p \cos \theta + E(p)}{M_{6q}} - x_{6q}\right), \quad (4)$$

where $E(p) = \sqrt{p^2 + m^2}$ with m as the quark mass, and M_{6q} is the mass of the six-quark bag, x_{6q} is the momentum fraction of the six-quark bag carried by a single quark and $x_{6q}M_{6q} = xM$ [7]. Note that $p \cos \theta$ is the third (z) component of the momentum, so that the plus component of the quark momentum is $E(p) + p \cos \theta$. We take $M_{6q} = 2M$ (its lowest possible value) to make a conservative estimate.

The term of interest $b_1(x)$ is given by

$$b_1^{6q}(x) = \frac{1}{2}(2)(F_0(x) - F_1(x))P_{6q}, \quad (5)$$

where P_{6q} is the product of the probability amplitudes for the 6-quark states to exist in the deuteron, and the factor of 2 enters because either state can be in the d -wave. Evaluation of F_H using Eq. (3) leads to the result:

$$b_1^{6q}(x) = -\sqrt{\frac{N_0 N_2}{2}} \frac{3}{4\pi} \int d^3p f_0 f_2 (3 \cos^2 \theta - 1) \delta\left(\frac{p \cos \theta + E(p)}{M} - x\right) P_{6q}. \quad (6)$$

To proceed further we specify the wave functions to be harmonic oscillator wave functions. We take $f_2(p) = -p^2 R^2 e^{-p^2 R^2/2}$, $f_0(p) = e^{-p^2 R^2/2}$, where R is the radius parameter. Within the present framework, the model is specified by only three parameters— R , the quark mass m and P_{6q} . The key question is whether such a model can reproduce the HERMES data point at $x = 0.452$ without using a value of P_{6q} large enough to conflict with conventional nuclear physics calculations that do not require a non-zero value. In other words, we ask if the hidden color states provide a substantial mechanism to make b_1 non-zero at large values of x . We note that the HERMES data has a large error bar, and our numerical results are chosen to reproduce the central value.

Our procedure is to adjust the value of P_{6q} to reproduce the data at that point and see how large a value is needed. To proceed further, we use a quark mass of 338 MeV. We expect that the 6-quark state should be somewhat larger than that of a nucleon, and therefore choose R to be 1.2 fm. We shall see that the calculations are not very sensitive to the exact value of R . The dependence on all three parameters is examined below.

The evaluation of $b_1^{6q}(x)$ proceeds by using $d^3p = 2\pi p^2 dp d\cos \theta$, integrating over $\cos \theta$, and changing variables to $u \equiv p^2 R^2$. The result is

$$b_1^{6q}(x) = \frac{6MR}{\sqrt{30\pi}} \int_{u_{\min}(x)}^{\infty} du e^{-u} \left[3((x^2 M^2 + m^2)R^2 + u - 2xMR\sqrt{u + m^2 R^2}) - u \right] P_{6q}, \quad (7)$$

where

$$u_{\min}(x) \equiv \frac{(x^2 M^2 - m^2)^2 R^2}{4x^2 M^2}. \quad (8)$$

3 Results

We now turn to the determine the contributions due to hidden color, b_1^{6q} provided by Eq. (7). The value of $b_1^{6q}(x = 0.452)$ is relevant because pionic and other contributions are negligible, and the measured value, $b_1 = -3.8 \pm 0.16 \times 10^{-3}$, differs from zero. We choose $P_{6q} = 0.0015$ to reproduce the central value using $R = 1.2$ fm and $m = 338$ MeV. The small value of P_{6q} shows that the 6 quark configuration has great impact on the computed value of b_1 . It is noteworthy that such a very, very small value can not be ruled out by any observations. Our value of P_{6q} is very small: small enough to evade any limits imposed by high accuracy conventional nuclear theory calculations of nucleon-nucleon scattering and deuteron properties that make no explicit reference to hidden color states. Our value of 0.0015 is too small to conflict with conventional nuclear calculations.

The results for b_1^{6q} are shown in Fig. 1. Results using the model parameters $R = 1.2$ fm, $m = 338$ MeV are shown as the solid curves in Figs. 1a and b, which also displays the sensitivity to the values of the parameters. There is relatively little sensitivity to the value of R , but more sensitivity to the value of m . However, there is wider dependence upon the choice of models. For example, if the HERMES point at $x = 0.452$ is not reproduced, this particular model will be ruled out. The exponential appearing

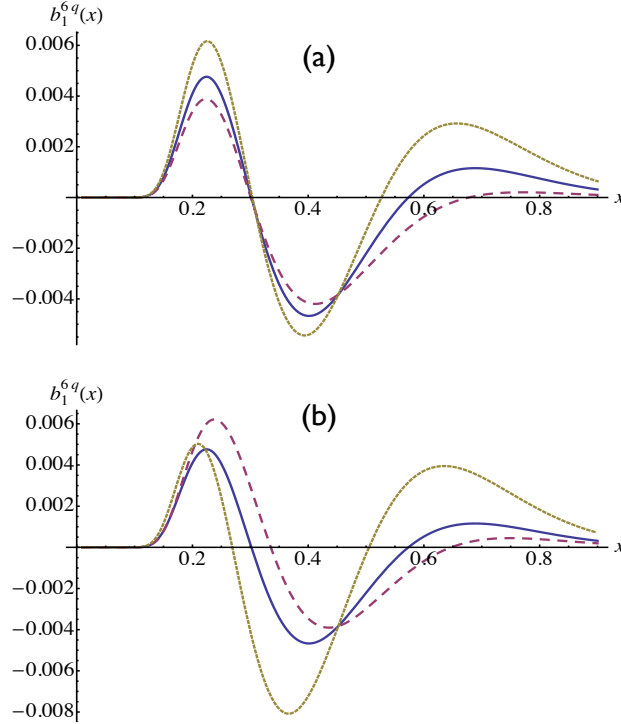


Fig. 1 (Color online) Computed values of b_1^{6q} from Eq. (7). Sensitivity to parameters is displayed. (a) Solid (blue) uses $R = 1.2$ fm, $m=338$ MeV, long dashed (Red) R is decreased by 10%, dotted (green) R is increased by 10%. (b) Solid (blue) uses $R = 1.2$ fm, $m=338$ MeV, long dashed (Red) m is increased by 10%, dotted (green), m is decreased by 10%. This figure is from Ref. [17].

in Eq. (7) renders the contribution very small for small values of x . This is seen in the figure. However, there is a large negative contribution at values of $x \approx 0.4$, as well as a double-node structure. The latter arises from the factor $3 \cos^2 \theta - 1$ appearing in the integrand of Eq. (6). The contributions of the hidden-color configurations are generally much smaller than those of exchanged pions except for values of x larger than about 0.35. We also predict that, for even larger values of x , b_1 changes sign and may have another maximum. This mechanism allows contributions at large values of x . A quark in a hidden color, six quark configuration can have up to two units of x . The parameter dependence of the model is also explored. Fig. 1a shows the dependence on the value of R and Fig. 1a shows the dependence on the value of m . For each of the curves P_{6q} is chosen so that the value at $x = 0.452$ is the same. Shifting the value of R while keeping $b_1^{6q}(0.452)$ fixed requires less than 4% changes in the value of P_{6q} . Increasing the value of the quark mass produces larger effects. Keeping $b_1^{6q}(0.452)$ fixed requires that the value of P_{6q} needs to be decreased by 20% if the value of the quark mass is increased by 10%, and the value of P_{6q} needs to be increased by about a factor of 1.8 if the value of the quark mass is decreased by 10%. In the remainder of this paper, we use the central values $R = 1.2$ fm, $m = 338$ MeV.

At this stage we can assess the size of our computed b_1^π and b_1^{6q} versus the only existing data [2]. These data is given in Table 1 along with our computed values of b_1^π using the pion structure functions of Re. [1] and the three modes of [20]. These modes differ in the fraction of momentum carried by the sea: 10%, 15% and 20% for modes 1, and 3 respectively. The differences obtained by using different structure functions are generally not larger than the experimental error bars. For values of x less than about 0.2, there is qualitative agreement between the measurements and the calculations of b_1^π (which are much larger than those of b_1^{6q}), given the stated experimental uncertainties and the unquantifiable uncertainty caused by lack of knowledge of the sea. However, the large-magnitude negative central value measured at $x = 0.452$ is two standard deviations away from the value provided by b_1^π but in accord with the value provided by b_1^{6q} . Thus our result is that one can reproduce the Hermes measurements by

Table 1 Measured values (in 10^{-2} units) of the tensor structure function b_1 . Both the statistical and systematic uncertainties are listed. The numbers in parenthesis refer to the structure function modes of Ref. [20] .

$\langle x \rangle$	$\langle Q^2 \rangle$ [GeV ²]	b_1 [10 ⁻²]	$\pm \delta b_1^{\text{stat}}$ [10 ⁻²]	$\pm \delta b_1^{\text{sys}}$ [10 ⁻²]	b_1^π [1] [10 ⁻²]	b_1^π [20] (1) [10 ⁻²]	b_1^π [20] (3) [10 ⁻²]	b_1^{6q} [10 ⁻²]
0.012	0.51	11.20	5.51	2.77	10.5	15.5	24.1	0.00
0.032	1.06	5.50	2.53	1.84	5.6	6.8	8.9	0.00
0.063	1.65	3.82	1.11	0.60	4.2	3.7	4.1	0.00
0.128	2.33	0.29	0.53	0.44	1.6	1.3	1.3	0.01
0.248	3.11	0.29	0.28	0.24	-0.55	.13	0.12	0.41
0.452	4.69	-0.38	0.16	0.03	-0.02	-0.02	-0.022	-0.38

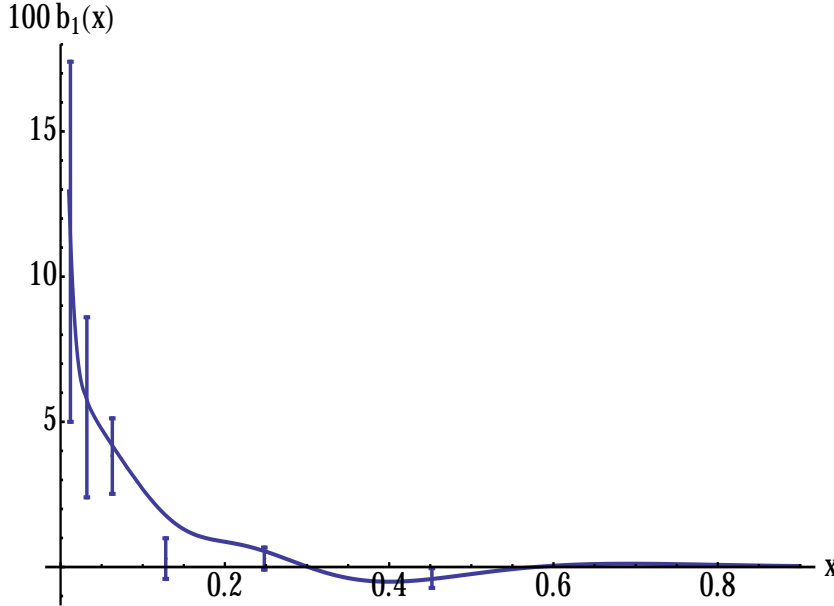


Fig. 2 Computed values of $b_1 = b_1^\pi + b_1^{6q}$. The pion structure function is that of [1], model 1. This figure is from Ref. [17].

using pion exchange contributions at low values of x and hidden-color configurations at larger values of x . Thus our result is that one can reproduce the Hermes measurements by using pion exchange contributions at low values of x and hidden-color configurations at larger values of x . This is also shown in Fig. 2, where very good agreement between data and our model can be observed. Please see [17] for more figures and details.

4 Summary

This paper contains an evaluation of the pion exchange and six-quark, hidden-color contribution to the b_1 structure function of the deuteron. The pion-nucleon form factor is constrained phenomenologically to reduce a possible uncertainty. There is some numerical sensitivity to using different pionic structure functions. The pionic mechanism is sizable for small values of x , and can reproduce Hermes data [2] for values of x less than 0.2. A postulated model involving hidden-color components of the deuteron is shown to complement the effects of pion exchange in reproducing the Hermes data for all measured values of x . This model is based on the accuracy of the Hermes data for its largest value of $x = 0.452$, and is chosen for simplicity. Many other models possible and we welcome further work to improve such models. Nevertheless, the availability of the Hermes data enables us to make predictions for an upcoming JLab experiment [19].

Acknowledgements This material is based upon work supported by the U.S. Department of Energy Office of Science, Office of Basic Energy Sciences program under Award Number DE-FG02-97ER-41014.

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